Improving Quantum Annealing through microcanonical thermalization

a one-dimensional study

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DARPA

RQMLS Project

Reversible/Quantum Machine Learning and Simulation

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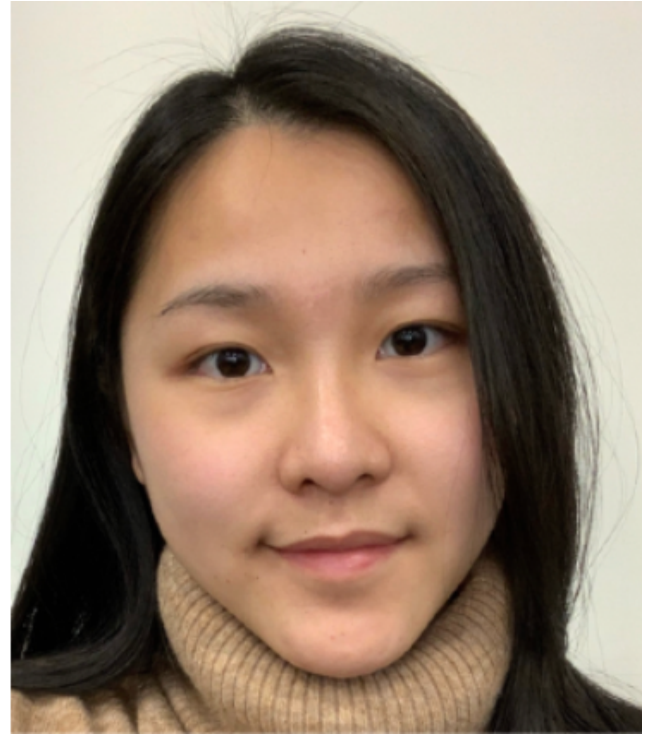

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Motivations: Pitfalls of Quantum Annealing

Bottleneck of "standard" QA: exponentially-small avoided crossings in the adiabatic path (*e.g.* first-order phase tr.)

Believed to be obiquitous in QA for finite-connectivity comb. opt. problems

Reverse Annealing/Population Transfer/etc.: use quantum dynamics to tunnel from a low-energy state to *new* low-energy states

New proposal: microcanonical thermalization through RFQA

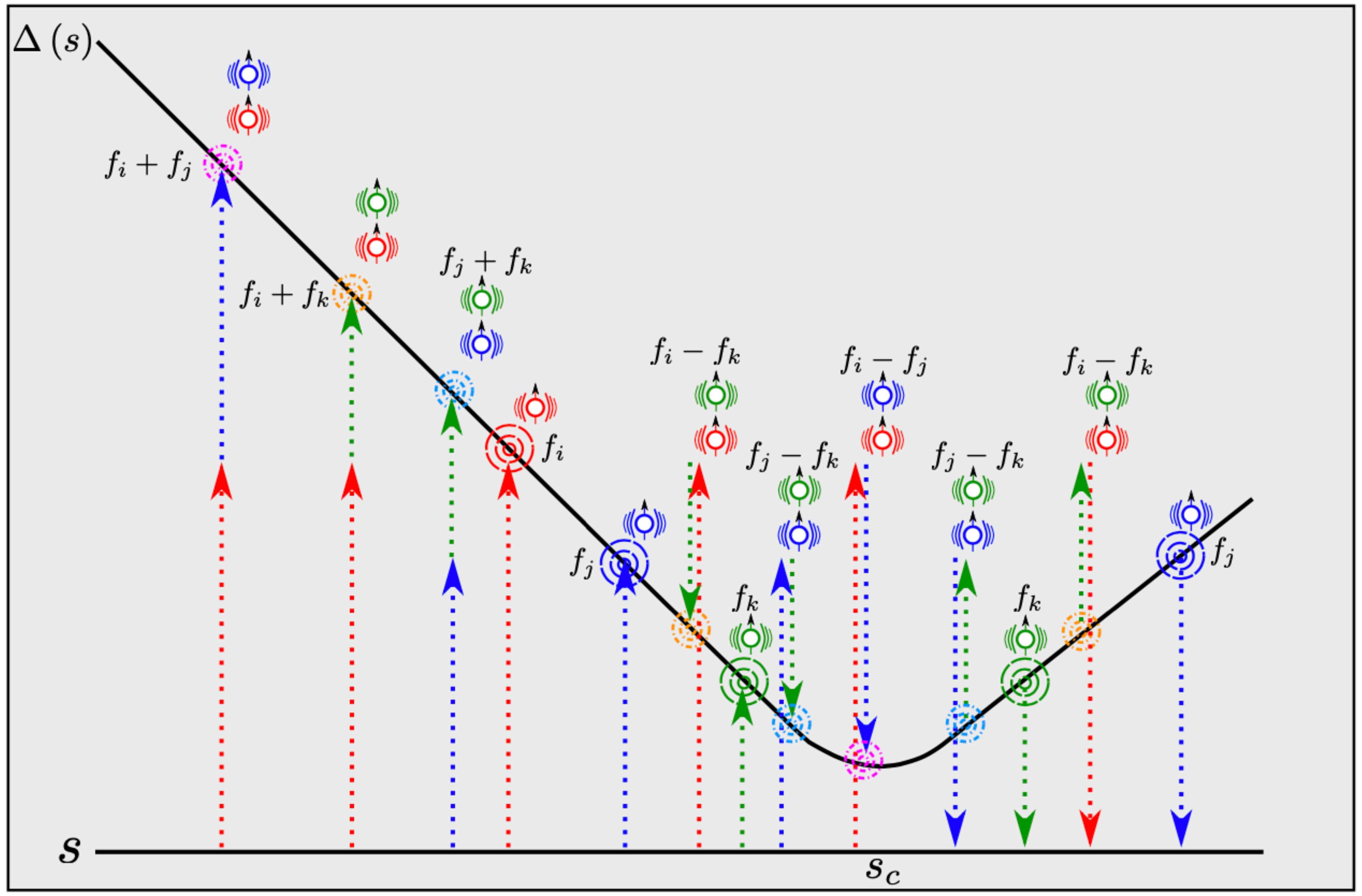
Additional Desiderata (for quantum speedup):

- generic mechanism ("problem-blind")
- not approachable by QMC methods
- noise-robust by design

“Microcanonical Thermalization”

induce a **diabatic** dynamics which close to an exp-small anticrossing populates

- **both states** approx. equally
- **faster** than *e.g.* uniform transverse field driver



RFQA driver: a uniform transverse field driver Hamiltonian is replaced by one where every transverse field oscillates independently at a randomly chosen frequency

Kapit and Oganesyan, QST 6, 025013 (2021)

By oscillating K single-spin operators, one can increase the tunnelling rate

$$r_{0\to 1} \propto \frac{\Omega_0^2}{W} \sum_{m=0}^{K} \Lambda^{2m} \binom{2K}{m} \propto \frac{\Omega_0^2}{W} (1 + 2\Lambda^2)^K$$

where Λ is a quantity which depends on the details of the system, but does **not** scale with the system size, or with K

Applications:

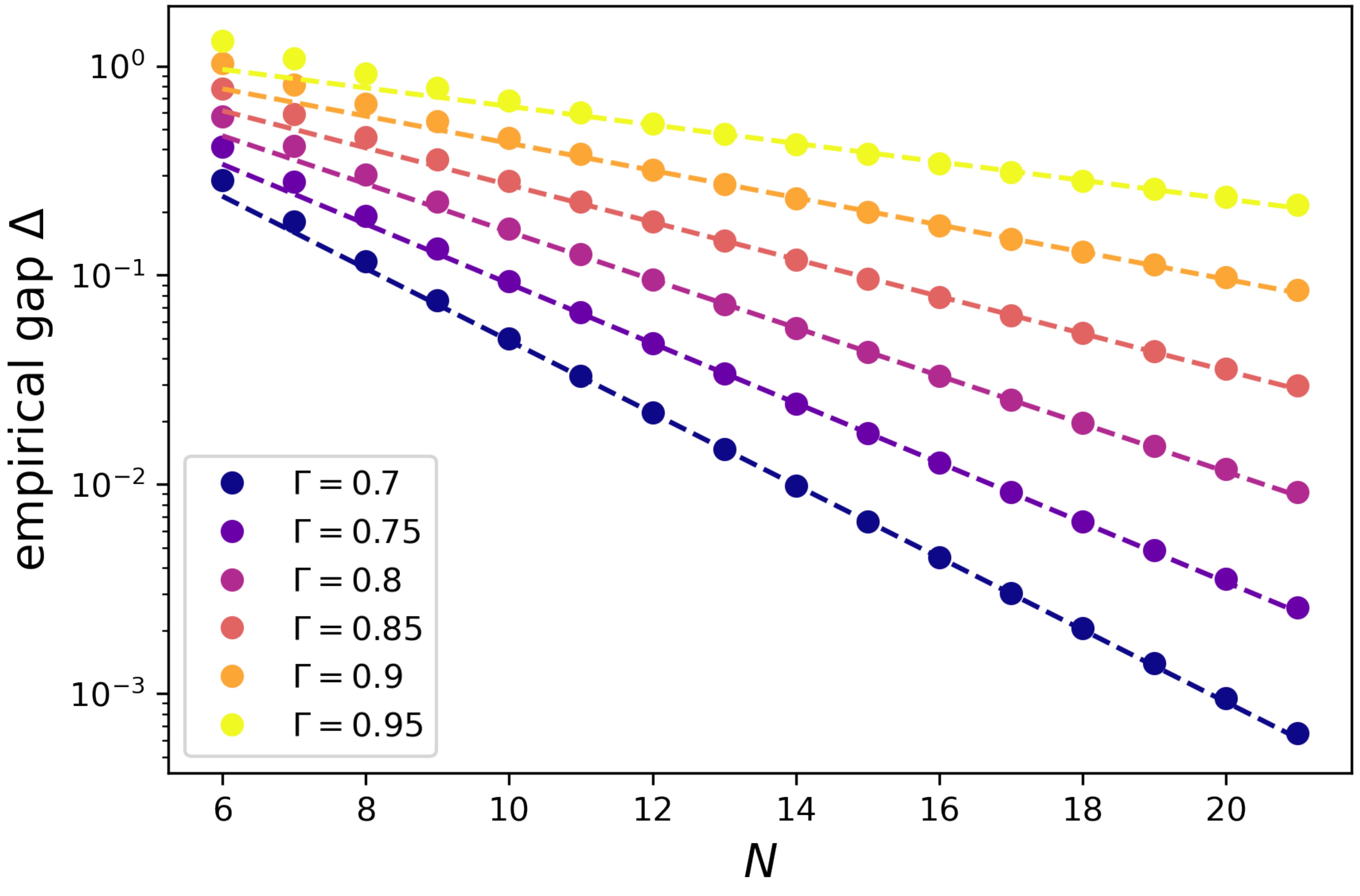
1) canonical QA approach (find the GS)
2) “fair sampling” (find multiple low-energy states given a low-energy state as a seed)

The Model: Transverse-Field Ising Chain

$$H = -J \sum_i \sigma_i^z \sigma_{i+1}^z - \Gamma \sum_i \sigma_i^x$$

Second-order Quantum Phase Transition at $\Gamma / J = 1$

Doubly-degenerate ground state for $0 < \Gamma < J$ **and** $N \to \infty$ **is split at finite** N **by an exponentially small gap**



MAIN GOALS OF THIS WORK:

1. Use the exponentially-small *empirical* gap between the quasi-degenerate ground states as a proxy for a "hard" avoided crossing in AQC
2. Study a simplified form of the chain freezing effect in an embedded problem

We will compare an **RFQA protocol** with a (uniform tr. field) **reverse-annealing-type protocol** as we move to

$$\Gamma \to \Gamma_c$$

The Protocols: Parameter Setting

$$H = -J \underbrace{\sum_i \sigma_i^z \sigma_{i+1}^z}_{H_p} - \underbrace{\sum_i h_i \sigma_i^z}_{\text{disordered fields}}$$

h_i are randomly chosen so that the energy splitting between the **two unperturbed ferromagnetic GSs** is "small"

$$E_+ - E_- \sim \text{unif}\Big([-\epsilon, \epsilon]\Big), \quad \epsilon = O(1)$$

Combinatorial "Problem":

Start in the initial state: $\Big|\uparrow \cdots \uparrow\Big\rangle$

GOAL: find the state: $\Big|\downarrow \cdots \downarrow\Big\rangle$

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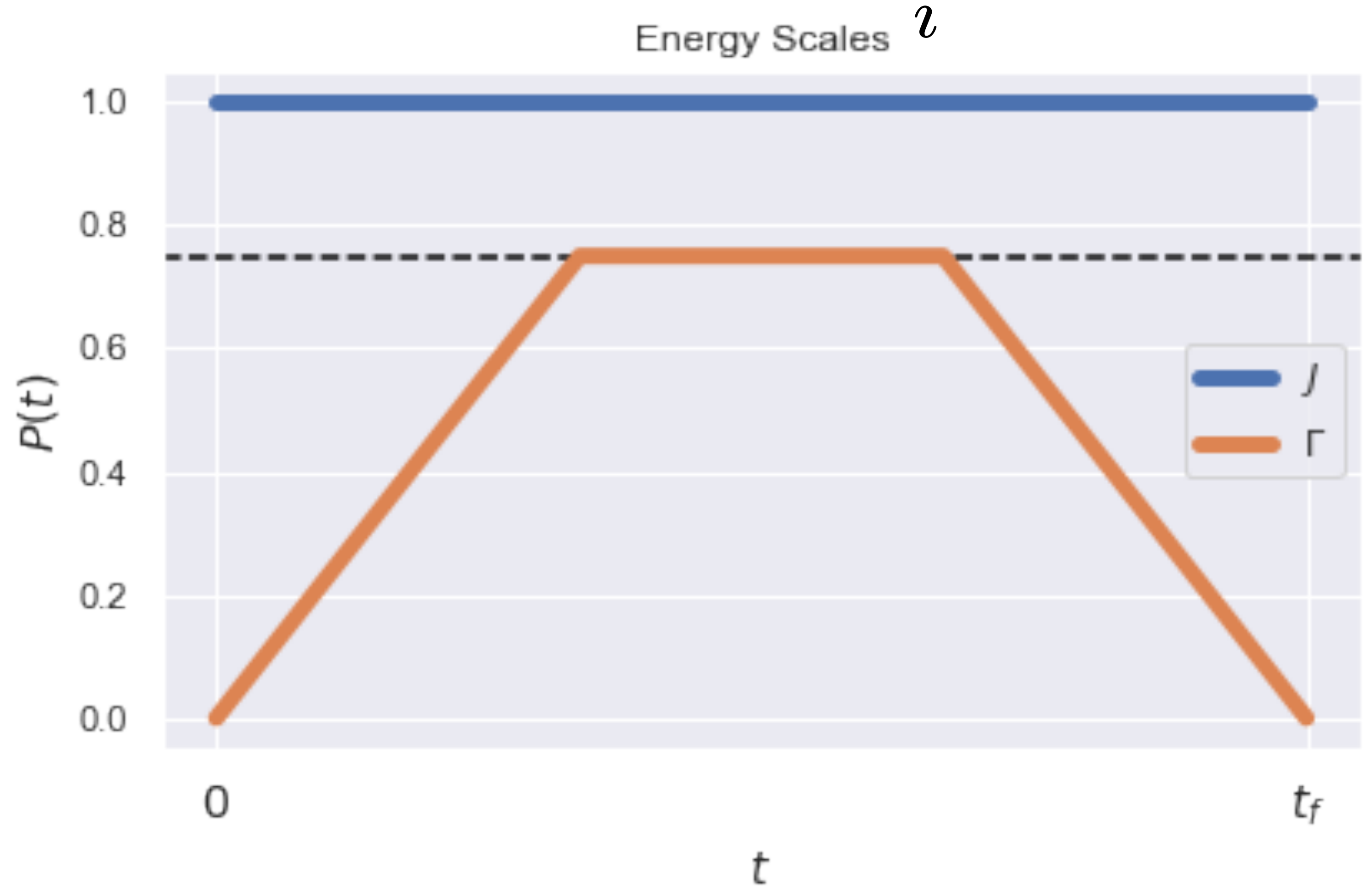
PT Driver

$$D_0(t) = -\Gamma(t) \sum_i \sigma_i^x$$



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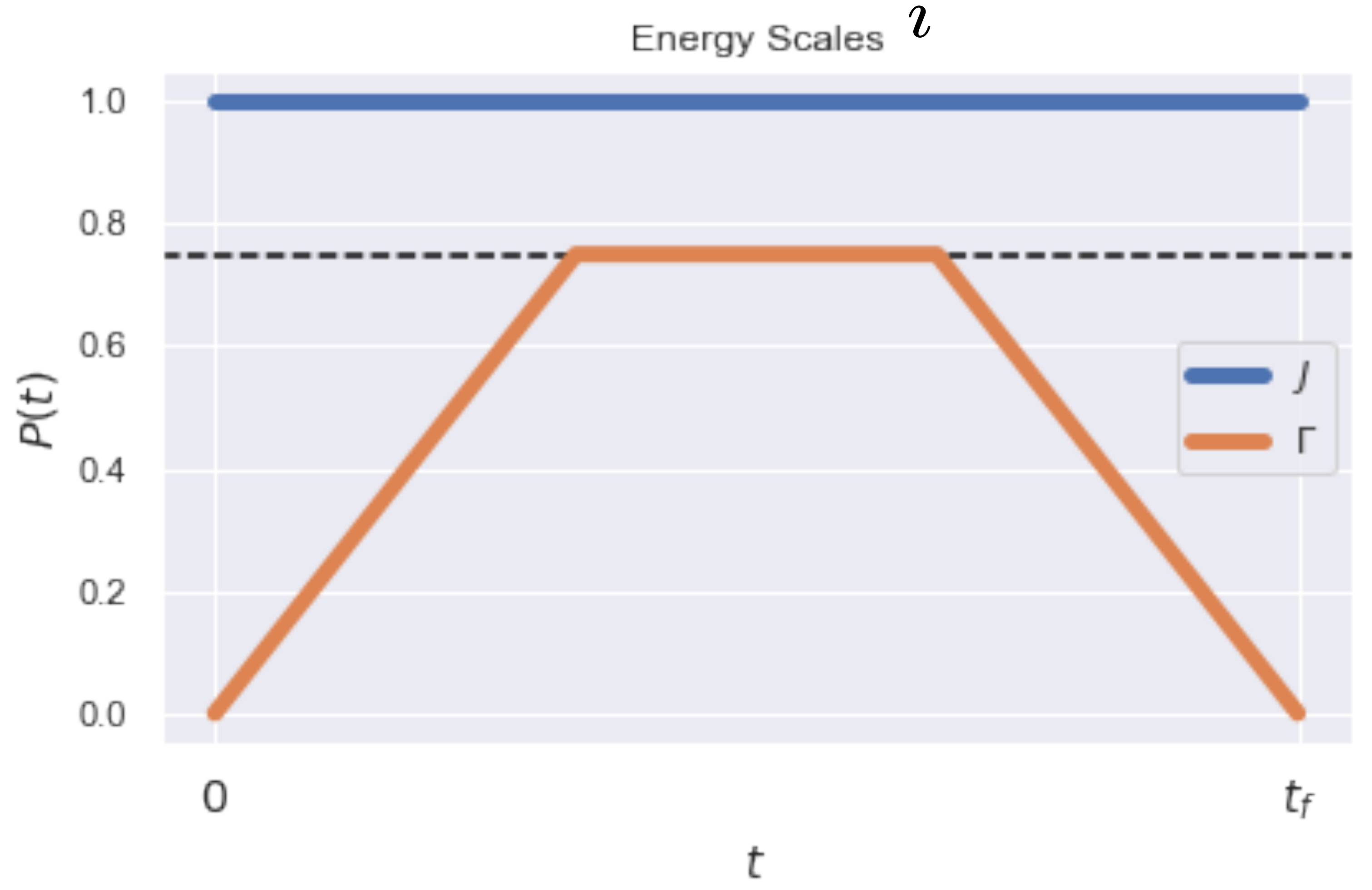

RFQA Driver

$$D_{RFQA}(t) = -\Gamma(t) \sum_j \Big[\cos\Big(\theta_j(t)\Big)\sigma_j^x + \sin\Big(\theta_j(t)\Big)\sigma_j^y\Big]$$

"windshield wiper" XY tr. field $\quad \theta_j(t) = \alpha \sin(2\pi\omega_j t)$

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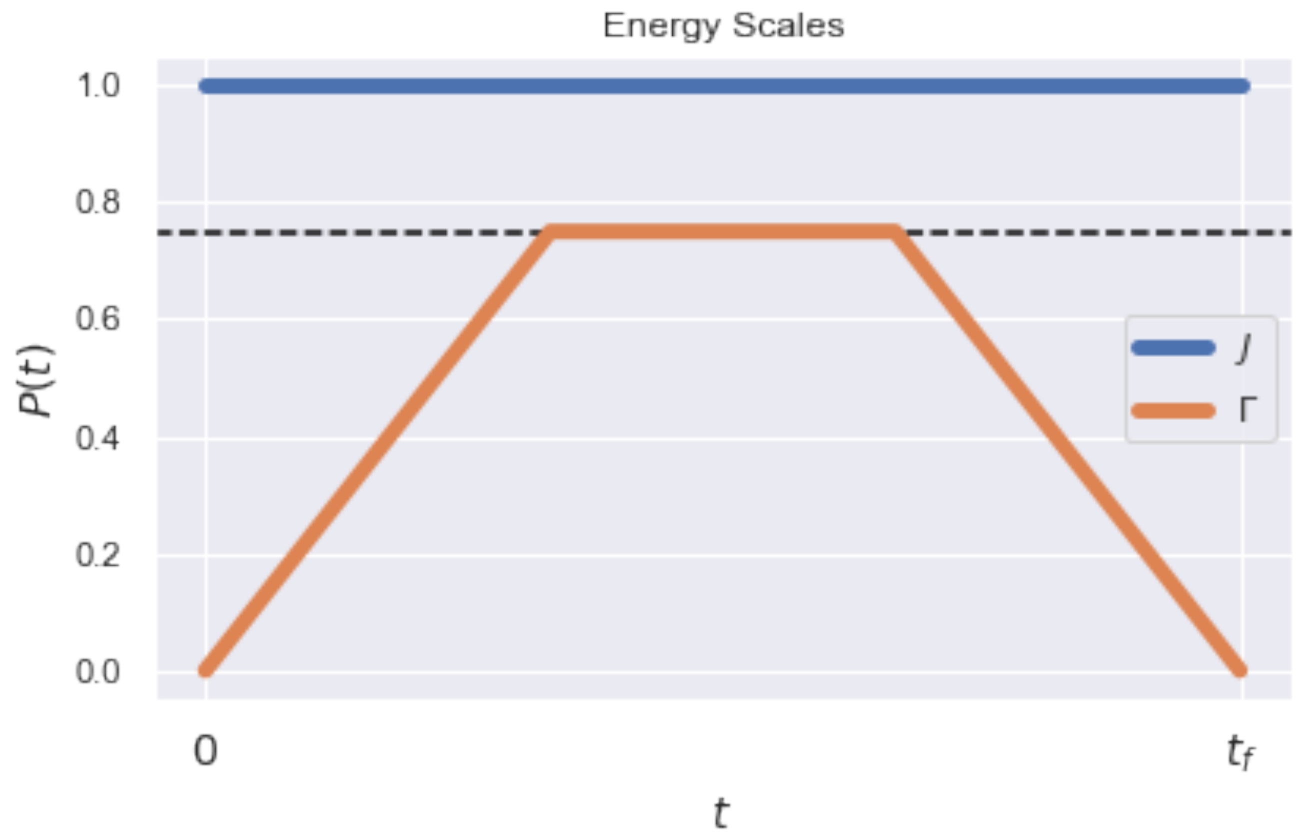

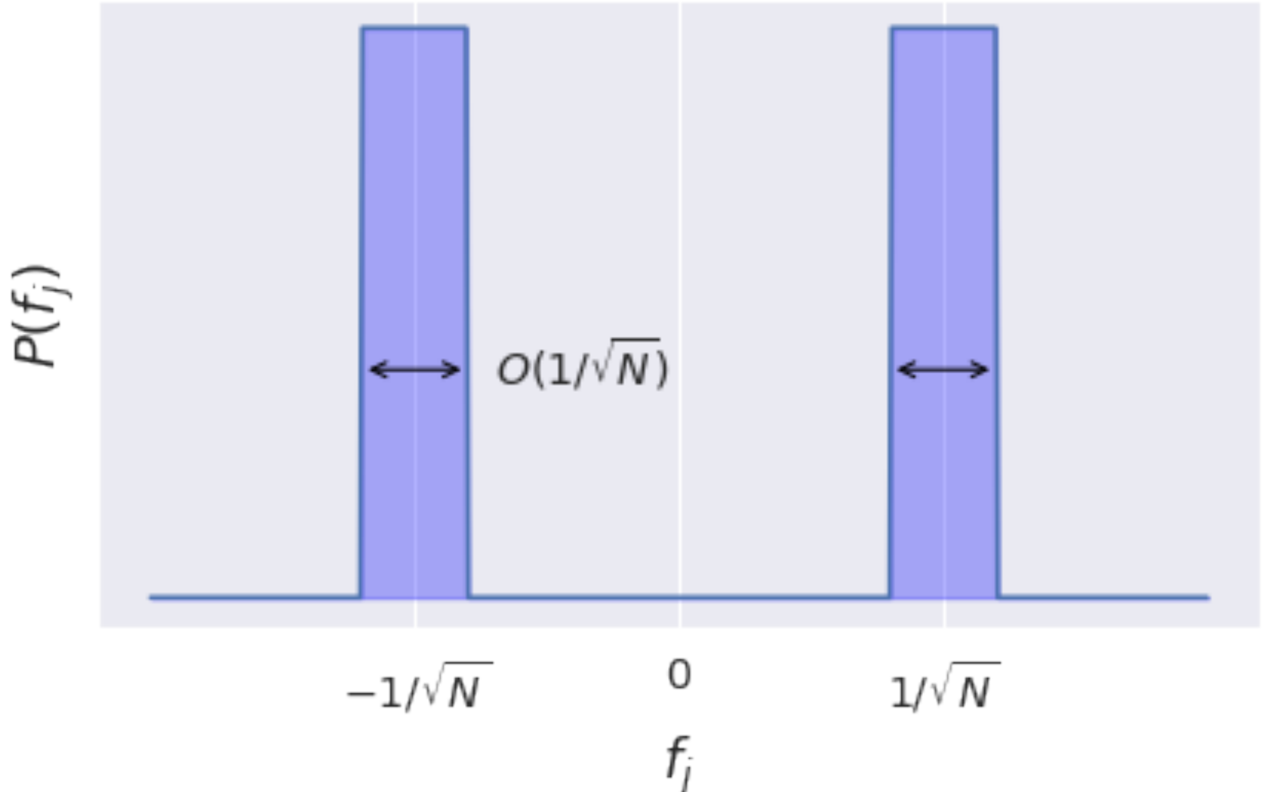

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"windshield wiper" XY tr. field $\quad \theta_j(t) = \alpha\sin(2\pi f_j t)$

$$f_j = \pm\omega_j, \quad \mathbb{E}[\omega_j] = \sigma[\omega_j] = O\Big(\frac{1}{\sqrt{N}}\Big)$$

"uncorrelated small frequencies" RFQA

PT vs RFQA Protocols

1) Prepare the ferromagnetic state

$$|\psi(0)\rangle = |\uparrow \cdots \uparrow\rangle$$

PT

RFQA

2) Dynamics
- Ramp up the driver to the desired intensity
- Wait for tunnelling
- Ramp the driver down to zero

$$|\psi(t_f)\rangle = U(t_f)|\uparrow \cdots \uparrow\rangle \qquad t_f = O(N)$$

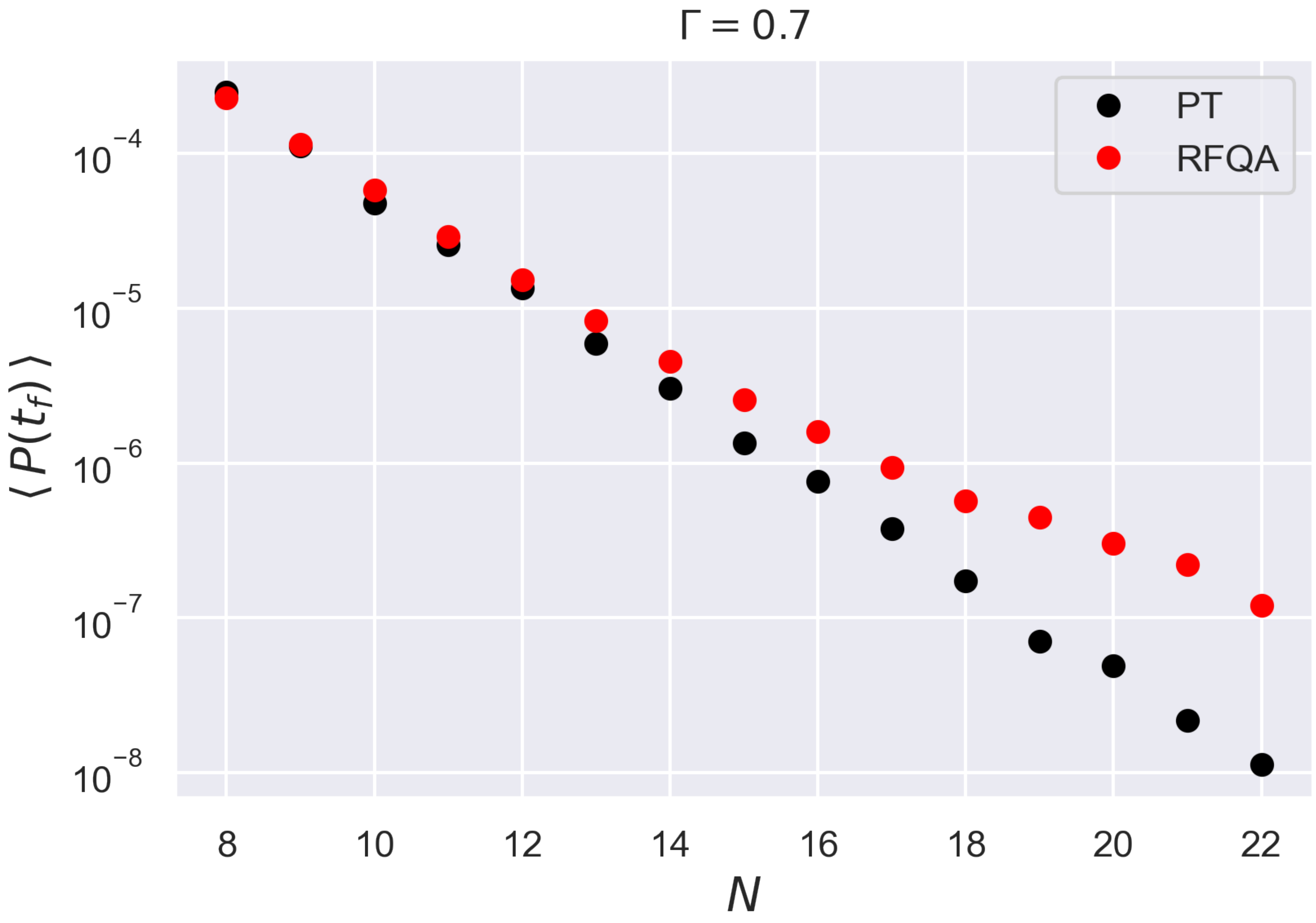
3) Measure the tunnelling probability to the target state

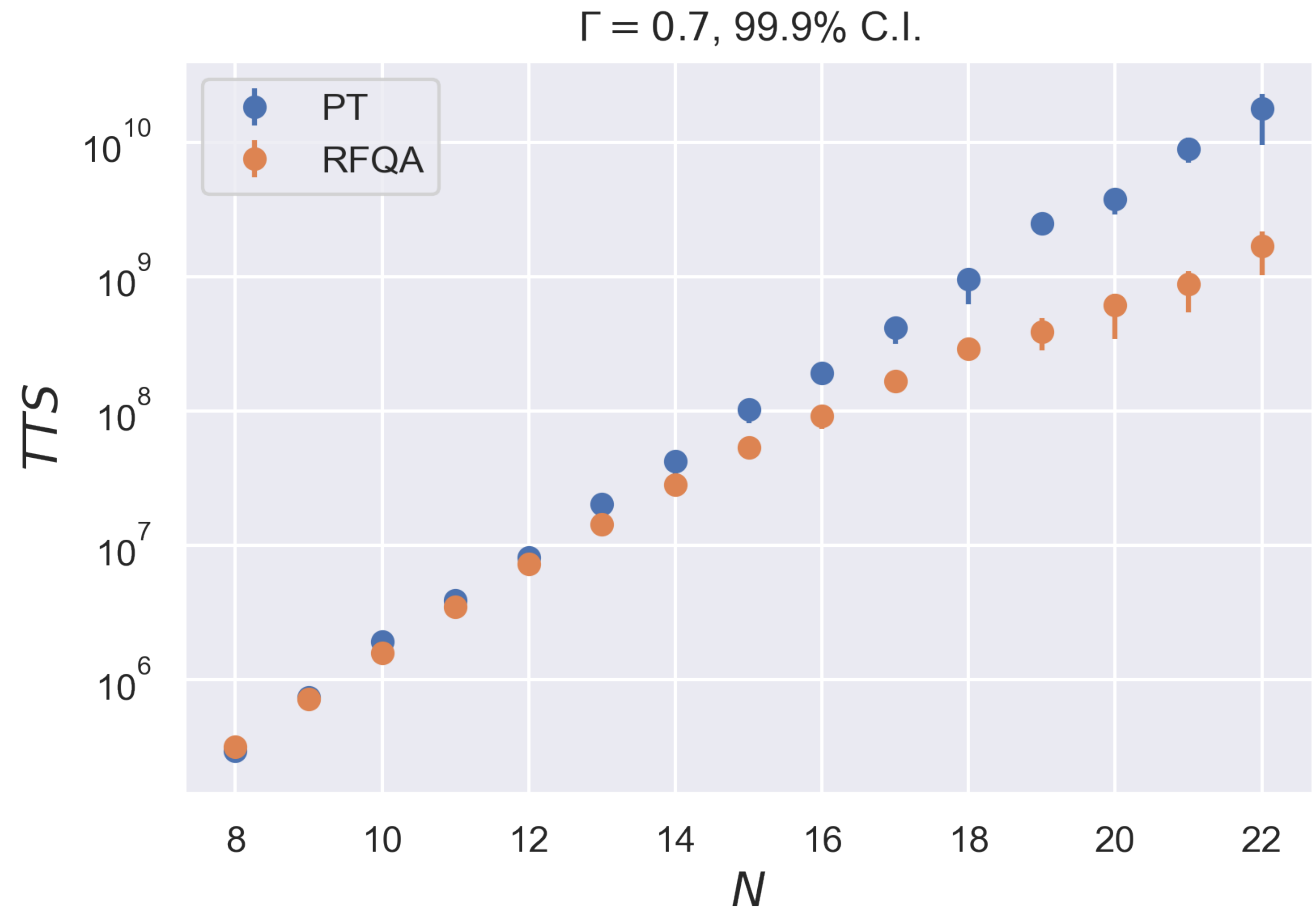
$$P(t_f) = |\langle\downarrow \cdots \downarrow|U(t_f)|\uparrow \cdots \uparrow\rangle|^2$$

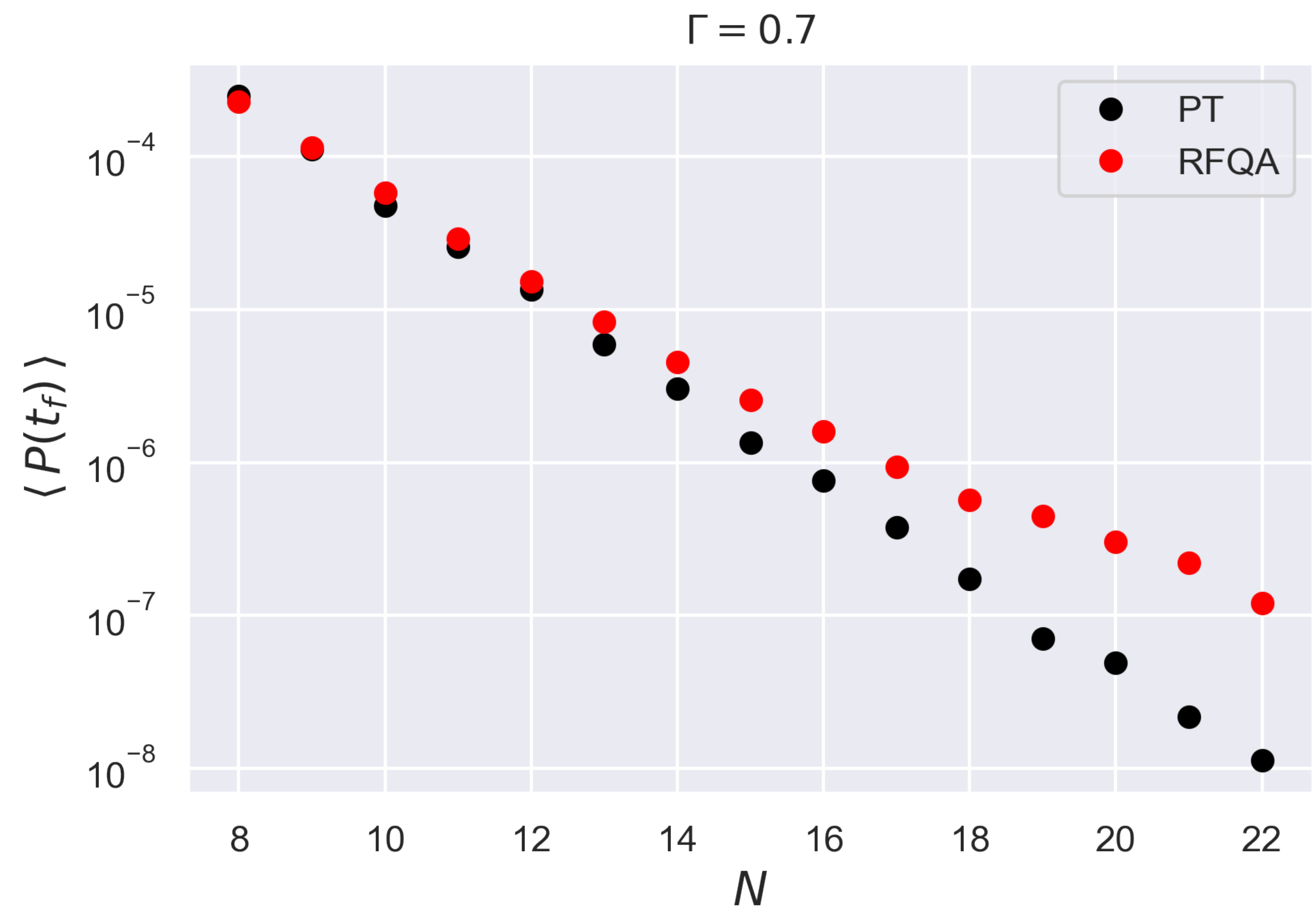
4) Repeat and compute the TTS

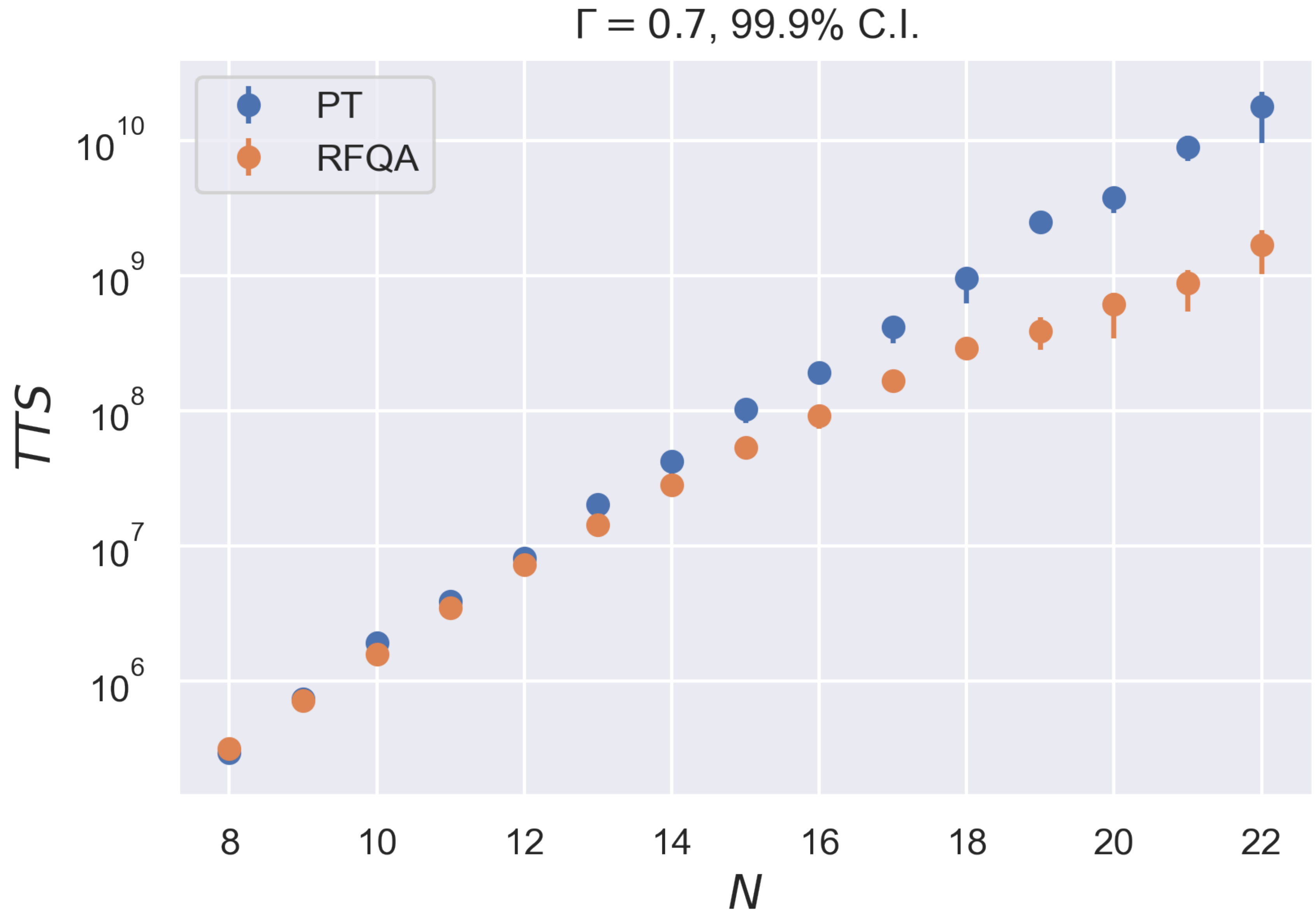
$$\mathrm{TTS} \equiv \frac{\log(1-0.99)}{\log(1-\langle P(t_f)\rangle)} \qquad \langle P(t_f)\rangle = \int P(t_f;\mathbf{f},\mathbf{h})\,\rho(\mathbf{f})\,\rho(\mathbf{h})\,\mathrm{d}\mathbf{f}\,\mathrm{d}\mathbf{h}$$

Generic behaviour for large enough Γ



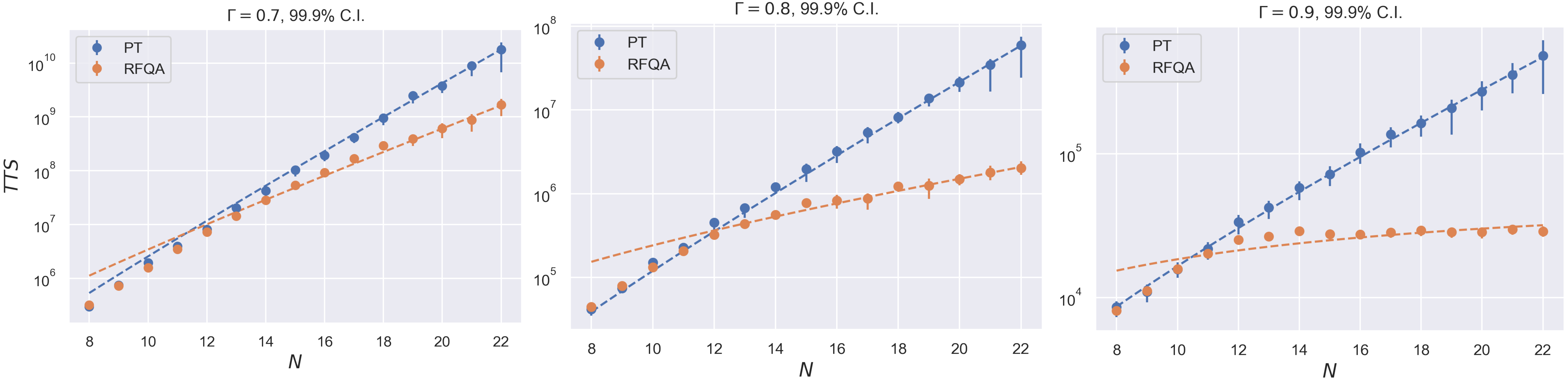






$$\Gamma \to \Gamma_c = 1$$

Exponential Fit: $TTS(N) = \beta N 2^{\alpha N}$



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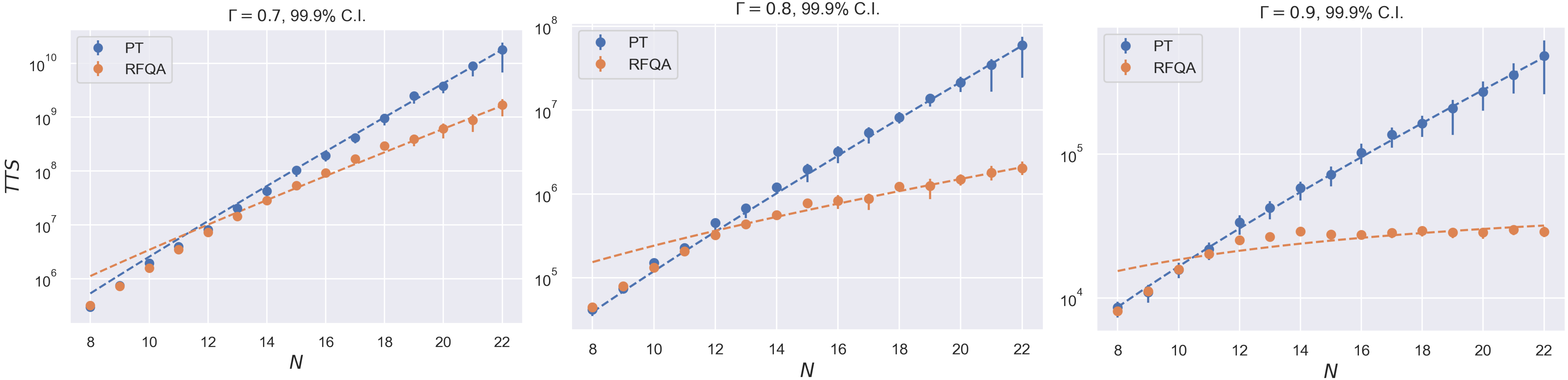

Γ	PT α exp.	RFQA α exp.	speedup
0.7	0.9(7)	0.7(1)	≈ 1.4
0.75	0.7(7)	0.3(3)	≈ 2.3
0.8	0.6(5)	0.1(6)	≈ 4.1
0.85	0.4(4)	0.0(4)	≈ 11.0
0.9	0.30(7)	-0.0(2)	?
0.95	0.12(7)	-0.0(9)	?

$$\frac{\alpha_{PT}}{\alpha_{RFQA}} = p$$

means

$\sqrt[p]{}$ **— speedup**

Conclusions

We've compared the RFQA and a uniform-field driver

- **RFQA exhibits a scaling advantage over a uniform-field driver**
- **the scaling advantage increases as** $\Gamma \to \Gamma_c$
- **for the system sizes studied, the TTS scaling exponent of RFQA is indistinguishable from zero close to the critical point of the QPT**
- **we expect this behaviour to generalize to finite-connectivity models (nothing in the protocols explicitly relies the 1-d structure of the model)**

Possible future directions

- **experimental implementation in open-system setting (MIT/LL)**
- **extend analysis to finite-connectivity combinatorial optimization problems**
- **better understand the behaviour close to criticality**

Thank You for Your Attention

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SUPPLEMENTARY MATERIAL

Numerical/Experimental Implementation: the Rotating Frame Picture

$$|\phi_R(t)\rangle \equiv R(t)|\psi(t)\rangle$$

$$R(t) \equiv \exp\left(i\sum_j \frac{\theta_j(t)}{2}\sigma_j^z\right)$$

$$\begin{aligned} i\frac{d}{dt}|\phi_R(t)\rangle &= i\dot{R}(t)|\psi(t)\rangle + R(t)i\frac{d}{dt}|\psi(t)\rangle \\ &= i\dot{R}(t)R^{-1}(t)|\phi_R(t)\rangle + R(t)H(t)R^{-1}(t)|\phi_R(t)\rangle \\ &\equiv H_R(t)|\phi_R(t)\rangle \end{aligned}$$

RFQA Effective Hamiltonian:

$$H_R(t) = -\frac{1}{2}\sum_j \frac{\partial\theta_j(t)}{\partial t}\sigma_j^z - J\sum_j \sigma_j^z\sigma_{j+1}^z - \sum_j h_j\sigma_j^z - \Gamma(t)\sum_j \sigma_j^x$$

Advantages of the rotating frame Hamiltonian:

- (Marginally) easier form
- Easy to implement on soon-to-be-available quantum hardware (S. Disseler @ MIT/LL)